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Transient thermoelastic contact problems for an elastic foundation

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Abstract

The paper presents a numerical solution of the problem of a hot rigid indenter sliding over a thermoelastic Winkler foundation at constant speed. It is shown analytically that no steady-state solution can exist for sufficiently high temperature or sufficiently small normal load or speed. The numerical solution shows that the steady-state solution, when it exists, is the final condition regardless of the initial conditions imposed. This suggests that the steady-state is also stable.

When there is no steady-state, the predicted transient behavior involves regions of transient stationary contact interspersed with regions of separation. Initially, the system typically exhibits a small number of relatively large contact and separation regions, but as time progresses, larger and larger numbers of small contact areas are established, until eventually the accuracy of the algorithm is limited by the discretization used.

1. Introduction

When two bodies slide against each other, frictional heating at the interface causes thermoelastic deformation which modifies the contact pressure distribution. Hills and Barber (1986) gave an analytical solution for sliding Hertzian contact, using a thermoelastic Green's function to reduce the problem to the solution of an integral equation with a Bessel function kernel. A remarkable feature of their results was that no steady-state solution could be found in certain ranges of the applied load and sliding speed without violation of the unilateral contact constraints. Similar results were demonstrated by Yevtushenko and Ukhanska (1993) for a problem with interfacial thermal contact resistance. Existence theorems can be proved for the corresponding transient problem, so we must conclude that in these parameter ranges the system must undergo periodic or random transient variations in contact

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conditions. Solution of the full transient thermoelastic contact problem is very difficult because the problem makes non-linear coupling between thermal and mechanical problem and in some cases leading to instabilities, so in the present paper, we consider the simplest system that exhibits the same steady-state characteristics—the sliding without friction of a hot rigid perfectly conducting indenter over a linear thermoelastic Winkler foundation. The Winkler foundation assumption could have affected the outcome of the formulations and calculation. However, it might be sufficient to determine the predicted transient behavior of contact regions.

2. Statement of the problem

Consider the problem illustrated in Fig. 1, where an indenter at temperature T_0 is pressed into the foundation with a force F and moves to the right at constant speed V. The mechanical behavior of the foundation is defined by the statement that the local contact pressure p is proportional to the local indentation u, i.e. $u(x,t) = cp(x,t)$, where c is the elastic foundation compliance. We also assume that lateral thermal conduction in the foundation can be neglected so that it behaves like a set of parallel one-dimensional rods oriented normal to the interface and each rod acts independently of its neighbours.[1]

If contact with the indenter is established at a particular point x at time $t = t_0$, the temperature for $y < 0, t > t_0$ is given by eqn (2.4.10) of Carslaw and Jaeger (1959) and the corresponding thermal displacement on the surface ($y = 0$) can be shown to be

$$\delta(x, t) = 2\alpha T_0\sqrt{\kappa(t - t_0)/\pi}, \tag{1}$$

where α, κ are respectively the coefficient of the thermal expansion and thermal diffusivity for the material. If contact at x ends at $t = t_1$, the thermal displacement will remain constant at the value $\delta(x, t_1)$ for $t > t_1$.

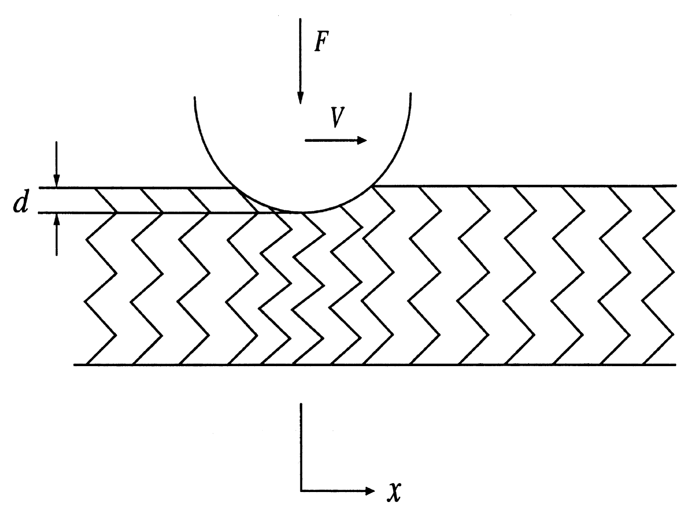

Fig. 1. Geometry configuration of transient thermal contact.

[1] This is quite a good approximation for the thermal behavior of a half-space if the Peclet number is sufficiently high.

Using these results, the gap functions can be defined as follows,

$$g(x,t) = g_0(x,t) - d(t) - \delta(x,t) + u(x,t), \tag{2}$$

where

$$g_0(x,t) = (x - Vt)^2/2R \tag{3}$$

is the gap between the indenter and an undeformed foundation and d is an unknown rigid body displacement which will generally vary with time.

The boundary condition for contact and separation regions can be written

$$\text{separation} \quad p(x,t) = 0; \quad g(x,t) > 0; \quad \text{contact} \quad p(x,t) > 0; \quad g(x,t) = 0 \tag{4}$$

and equilibrium requires that

$$F = \int_C p(x,t)\mathrm{d}x, \tag{5}$$

where C is the contact region. In the formulation of governing equation and boundary condition for contact and separation, the unknowns are $d(t)$ and $p(x,t)$. The problem is solved for contact and separation region when time evolves. Additionally $d(t)$ is also found at each time step.

3. Dimensionless formulation

The number of independent parameters can be reduced by utilizing the self-similarity of the punch profile. There are two length scales in the problem: (1) the radius R and (2) a force-related quantity $L = \sqrt[3]{cFR}$. We define the dimensionless coordinates $\hat{x} = x/L, \hat{t} = Vt/L$ and other dimensionless quantities through $\hat{\delta} = \mathrm{R}\delta/L^2, \hat{g} = Rg/L^2, \hat{d} = Rd/L^2, \hat{p} = cRp/L^2$. Introducing these definitions into eqns (1, 2, 3, 5) yields

$$\delta(\hat{x},\hat{t}) = \sqrt{\frac{3\lambda}{2}}\sqrt{\hat{t} - \hat{t}_0(\hat{x})}; \quad \hat{t}_0(\hat{x}) < \hat{t} < \hat{t}_1(\hat{x}) \tag{6}$$

$$\hat{g}(\hat{x},\hat{t}) - \hat{p}(\hat{x},\hat{t}) = \frac{(\hat{x} - \hat{t})^2}{2} - \hat{d}(\hat{t}) - \delta(\hat{x},\hat{t}), \tag{7}$$

and

$$\int_{\hat{C}} \hat{p}(\hat{x},\hat{t})\mathrm{d}\hat{x} = 1, \tag{8}$$

where $\lambda \equiv 8\alpha^2 T_0^2 \kappa R/(3\pi cFV)$.

Notice that with this formulation, the only dimensionless parameter governing the evolution of the process is λ which can be seen as a ratio between thermoelastic and elastic effects.

The contact boundary conditions (4) show that at least one of $\hat{g}$, $\hat{p}$ must be zero for all $\hat{x}$ and that the other cannot be negative. Thus, if the right-hand side of eqn (7) can be calculated, a positive value will indicate a state of separation and will be equal to the local value of $\hat{g}$, whereas a negative value will correspond to contact and will be equal to the local value of $(-\hat{p})$.

4. Steady-state solution

Since the contacting body moves at constant speed, it is natural to expect the long-time behavior to become invariant in a frame of reference moving with the body. In particular, the indentation $\hat{d}$ would then be independent of $\hat{t}$. Denoting the value of this constant by d_0, we can then find the leading edge $\hat{a}(\hat{t})$ of the contact area by enforcing $\hat{g}=0, \hat{p}=0$ in eqn (7), with the result

$$\frac{(\hat{a}-\hat{t})^2}{2} = d_0, \tag{9}$$

since the thermal expansion must be zero before contact starts. It follows that $\hat{a}(\hat{t}) = \sqrt{2d_0} + \hat{t}$, or alternatively that $\hat{t}_0(\hat{x}) = \hat{x} - \sqrt{2d_0}$.

The expansion in the contact area can now be calculated from eqn (6) and the contact pressure from (7), with the result

$$\hat{p}(\hat{x}, \hat{t}) = -\frac{(\hat{x}-\hat{t})^2}{2} + d_0 + \sqrt{\frac{3\lambda}{2}}\sqrt{\hat{t} - \hat{x} + \sqrt{2d_0}}. \tag{10}$$

The trailing edge of the contact area $\hat{b}(\hat{x})$ is defined by the condition that the contact pressure goes to zero. One solution of the resulting equation is clearly $\hat{a}(\hat{t})$ and the other is the one real root of the cubic equation $(\sqrt{2d_0} + \hat{t} - \hat{b})(\sqrt{2d_0} - \hat{t} + \hat{b})^2 = 6\lambda$. Once $\hat{a}, \hat{b}$ have been determined, the corresponding value of λ can be obtained from eqn (8).

The special case is where $d_0 = 0$ has a simple solution, since we then have $\hat{a}(\hat{t}) = \hat{t}$ from (9) and $\hat{b}(\hat{t}) = \hat{t} - \sqrt[3]{6\lambda}$. The corresponding pressure distribution is

$$\hat{p}(\hat{x}, \hat{t}) = -\frac{(\hat{x}-\hat{t})^2}{2} + \sqrt{\frac{3\lambda}{2}}\sqrt{\hat{t} - \hat{x}}, \tag{11}$$

and eqn (8) then yields

$$1 = \int_{\hat{b}}^{\hat{a}} \hat{p}(\hat{x}, \hat{t})\mathrm{d}\hat{x} = \int_0^{\sqrt[3]{6\lambda}} \left(\sqrt{\frac{3\lambda}{2}}\sqrt{\hat{y}} - \frac{\hat{y}^2}{2}\right)\mathrm{d}\hat{y} = \lambda, \tag{12}$$

where $\hat{y} = \hat{t} - \hat{x}$.

Only positive values of d_0 are admissible and it can be shown that the integral in (8) is a monotonically increasing function of d_0 in the range $d_0 > 0$. Thus, there is no steady-state solution of the assumed form if $\lambda > 1$. In other words, steady solutions do not exist at sufficiently large values of temperature difference or sufficiently small values of force or velocity.

To determine how the system behaves at large values of time for $\lambda > 1$, a numerical solution of the problem has been developed, which is described in the next section.

5. Numerical implementation

The contact problem can be discretized in space and time by dividing the elastic foundation into discrete strips of width $\Delta\hat{x}$ and proceeding in increments of time $\Delta\hat{t}$.

It is convenient to take the vertical rigid body displacement $\hat{d}(\hat{t}_j)$ as a fundamental variable defining the evolution of the process, where $\hat{t}_j$ is the time after the j-th time increment. If $\hat{d}(\hat{t}_j)$ were known for all j, the trajectory of all points on the moving body would also be known and hence we would be able to determine the time $\hat{t}_0(\hat{x}_i)$ at which any given element at $\hat{x}_i$ comes into contact. The subsequent thermal expansion could then be determined for each $\hat{x}_i$ from eqn (6) and the contact pressure $\hat{p}(\hat{x}_i, \hat{t}_j)$ from eqn (7). A negative value of $\hat{p}(\hat{x}_i, \hat{t}_j)$ at any contacting element indicates loss of contact and could be used to set the value for $\hat{t}_1(\hat{x}_i)$.

Of course, $\hat{d}(\hat{t}_j)$ is not known a priori. Instead, it must take whatever value is required to satisfy the equilibrium condition (8), which in discretized form can be written

$$S \equiv \sum_{i \in \hat{C}} \hat{p}(\hat{x}_i, \hat{t}_j) \Delta \hat{x} = 1, \tag{13}$$

where $\hat{C}$ is the set of nodes in contact. The relation between S and $\hat{d}$ is non-linear because the contact area $\hat{C}$ varies with $\hat{d}$. In the numerical solution, we must therefore determine $\hat{d}$ at each time step by iteration. We take the value of $\hat{d}$ at the previous time step as an initial guess for this process. The right-hand side of (7) can then be calculated for all nodes and those in which negative values are obtained correspond to contact nodes, which make a contribution to the sum in eqn (13). The value of S so calculated will generally differ from unity and we therefore make a correction to $\hat{d}$ using the algorithm

$$\hat{d}_{\text{new}} = \hat{d}_{\text{old}} + \frac{1 - S}{N_C \Delta \hat{x}}, \tag{14}$$

where N_C is the number of elements in $\hat{C}$ at the previous iteration.

Equation (13) shows that this would yield the correct value of $\hat{d}$ in one iteration if the elements of $\hat{C}$ were unchanged after the iteration. Of course, this is not generally the case, but convergence is found to be very rapid and terminates completely once the increment in $\hat{d}$ is small enough to have no further effect on the set of contact nodes.

When the value of $\hat{d}(\hat{t}_j)$ has been established, the elements are scanned to determine which, if any, change state from separation to contact or vice versa during the j-th time step, in order to set the corresponding value of $\hat{t}_0$, $\hat{t}_1$ in eqn (6). The time can now be updated through

$$\hat{t}_{j+1} = \hat{t}_j + \Delta \hat{t} \tag{15}$$

and the process repeated indefinitely.

6. Results

The results confirm that for $\lambda < 1$ the system settles into a steady state after an initial transient period. This demonstrates that the steady-state solution is stable under transient perturbations. Figure 2 shows the extent of the contact area and the rigid body penetration $\hat{d}$ as functions of time $\hat{t}$ for $\lambda = 0.9$. The indenter is assumed to be pressed against the foundation at $\hat{t} = 0$ and to start moving immediately at speed V. In the initial transient, the leading edge of the contact area remains unchanged, whilst the trailing edge moves, reducing the total extent of contact. During this period, thermal expansion forces the bodies apart, causing $\hat{d}$ to decrease. Eventually the expansion levels off and the additional elastic displacement associated with the reduction in contact area (and consequent increase in contact pressure) allows $\hat{d}$ to increase again, until a new separated contact area is established. A steady state, with a single contact area, both boundaries of which move at speed V, is established after about $\hat{t} = 5$.

As λ approaches unity, the duration of the initial transient increases and involves a succession of separated contact areas and oscillations in the value of $\hat{d}$. For values greater than unity, regions of

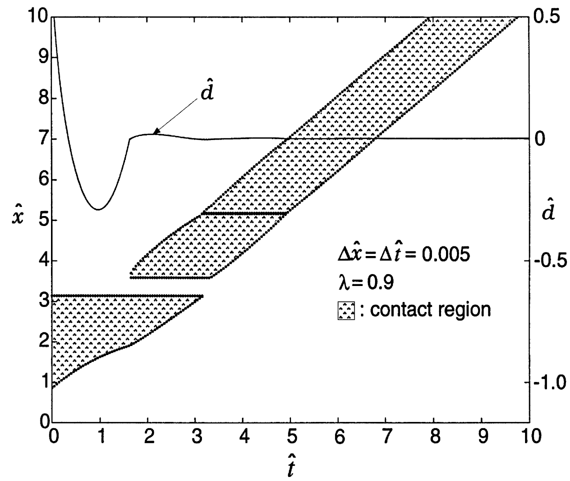

Fig. 2. Extent of contact area and rigid body penetration $\hat{d}$ as functions of time $\hat{t}$ for $\lambda = 0.9$.

alternating contact and separation occur for all time and the size of the typical contact area appears to decrease continually as the state evolves.

Figures 3 and 4 show results for $\lambda = 1.5, 6$, respectively. For Fig. 3 in the early stage, the same explanation given for Fig. 2 can be used, and then its state is best characterized as contact with numerous small intervening regions of separation, after about $\hat{t} = 3$. A corresponding small oscillation occurs in the rigid body penetration $\hat{d}$.

For $\lambda = 6$, large separation zones alternate with relatively small zones of contact. Periodically the penetration $\hat{d}$ increases sufficiently for a new contact area to form (and hence expand) or lateral motion permits the trailing contact zone to be lost.

This transient state does not appear to tend to a steady periodic state (e.g. one with equal-spaced contact areas). Instead, contact areas appear randomly grouped, often in clusters. Also, the typical

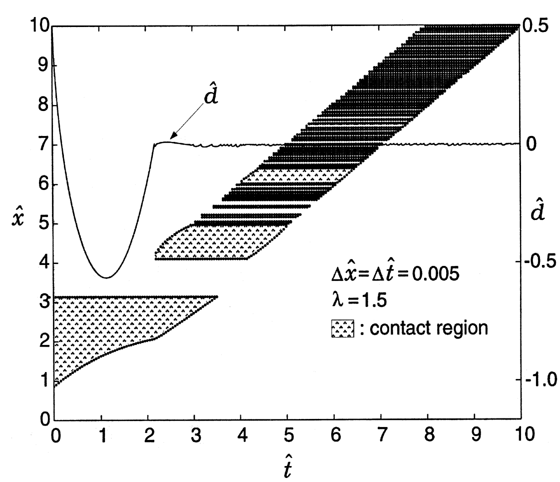

Fig. 3. Extent of contact area and rigid body penetration $\hat{d}$ as functions of time $\hat{t}$ for $\lambda = 1.5$.

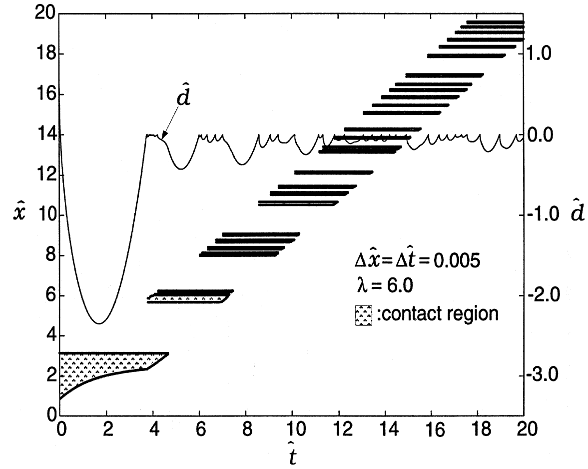

Fig. 4. Extent of contact area and rigid body penetration $\hat{d}$ as functions of time $\hat{t}$ for $\lambda = 6.0$.

contact area size decreases as time progresses, ultimately approaching the level of discretization of the algorithm.

7. Discussion and conclusions

This investigation presents a numerical solution to the problem of a hot rigid indenter sliding over a thermoelastic Winkler foundation at a constant speed. The numerical solution shows that the steady-state solution, when it exists, is the final condition regardless of the initial conditions imposed. This suggests that the steady state is also stable. When there is no steady state the predicted transient behavior involves regions of transient stationary contact interspersed with regions of separation. Initially, the system typically exhibits a small number of relatively large contact and separation regions, but as time progresses, larger and larger numbers of small contact areas are established, until eventually the accuracy of the algorithm is limited by the discretization used. This study also shows that for sufficiently high temperature, low speed, or applied force, a moving indenter will not slide steadily over a thermoelastic foundation, but instead will ride on a series of stationary thermoelastically generated corrugations. Indications are that similar results will apply in the case of thermoelastic half-space.

A question exists as to how much the Winkler foundation assumption, which states that the local contact pressure p is proportional to the local indentation, may affect the predicted contact behavior. One of the limitations of the Winkler foundation assumption is that it does not consider the shear effect. However, the assumption does not affect the main characteristics which determine if a steady state solution will be achieved. As mentioned in the Introduction, Hills and Barber (1986) have shown that no steady-state solution can occur for certain ranges of the applied load and sliding speed without violation of the unilateral contact constraints. The numerical results obtained in the present study confirm their predicted phenomena; for $\lambda > 1$, contact condition will be periodic or randomly transient. Recall that the parameter $\lambda(= 8\alpha^2 T_0^2 \kappa R/(3\pi cFV))$ represents the thermoelastic and elastic effects. In the Winkler foundation model, λ is directly related to c, the elastic foundation compliance. For example, the compliance value is small for a hard material. Thus, λ is larger for a hard material than for a soft material. So for a harder material, there is a greater chance that a steady-state solution will not be achieved. On the other hand, our calculations may not predict the exact transient behavior of real thermoelastic surfaces, but the calculations provide an indication that the periodic or random contact will occur under a certain range of applied load, sliding speed, and temperature. However, no progress has been reported on this challenging unilateral boundary value problem.

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References

Carslaw, H., Jaeger, J.C., 1959. The Conduction of Heat in Solids, 2nd ed. Clarendon Press, Oxford.

Hills, D.A., Barber, J.R., 1986. Steady sliding of a circular cylinder over a dissimilar thermally conducting half-plane. Int. J. Mech. Sci. 28, 613–622.

Yevtushenko, A.A., Ukhanska, O.M., 1993. Plane contact problem of thermoelasticity with quasi-steady frictional heating. Int. J. Engng. Sci. 31, 1565–1573.